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Applications of Graph Theory in Engineering curriculum

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Abstract— The Study of planar graphs necessarily involves the topology of the plain. It is especially relevant in the study of planar graphs are those which deal with the curves. In terms of Jordan curves it is continuous non self intersecting curve and whose origin and terminus coincide. In general properties of Jordan curves come into play in planar graph theory. If J be the Jordan Curve in the plan the remaining of the plane indicates two disjoint open sets called interior and exterior of Jordan(J). In this Application we can observe that the Jordan(J)= Int.J Ω Ext.J. Jordan curve theorem states that the line joining a point in Int.J to a point in Ext.J must meet J in some point in its planar graph.

Keywords—toplogy, Jordan curves, Self intersecting curve, interior and exterior, coincide.

1. Introduction

A graph which can be drawn on a plan without any cross overs is said to be a planar or a graph is said to be embeddable in the plane if it can be drawn in the plane so that its edges coincides only at their ends. Study of planar graphs necessarily involves the topology of the plane. The results of topology that are especially relevant in the study of planar graphs are those which deal with Jordan curves. A Jordan curve is a continuous non-self intersecting curve whose origin and terminus coincide. We can verify the same conditions by using the non planarity of Kn.

If we consider J be a Jordan curve in the given plane. Then the remaining of the plane is divided into two disjoint open sets called the interior and exterior of Jordan(J).

A connected part of a plane which does not contain any vertices and is surrounded by edges is called a region of planar embedding. The part out side the embedding is known as region or exterior region of the given graph. The vertices surrounding a region are called boundary vertices and the edges surrounding are called boundary edges. We can observe that in the research process some fundamental properties of planar embedding of graphs.

As a fundamental property we can observe that in any Planar Graph extension If a planar embedding of a connected graph G then it has n vertices, m edges and ‘ r’ regions. We can also indicate this as r+n=m+2.

Similarly in a Linear Bound If a simple connected planar graph G has more than or equal to three(3) vertices and ‘m’ edges then ‘m’ is less than or equal to(3n-6).
We can also observe that The Minimum Degree Bound for a simple planar graph G, δ (G) $\leq$ 5.

As an additional conditions of Planar graphs we can notify the condition of Kuratowski’s Principle A graph is planar if and only if none of its sub graphs can be transformed to K5, or K3,3 by contracting edges.

2. Euler’s Polyhedron Conditions

If a planar embedding of a connected graph G has n vertices m edges and r regions, then r+n =m+2.
If we use induction on ‘m’.

Induction Basis: m=0. Planar embedding of G has only one vertex and one region that is exterior region which is said to be Universal true statement.

Induction Hypothesis: Assuming that the condition or the given principle is true for m $\leq$ l. (l $\geq$ 0).

Induction Statement :“The stated condition is true for m= l+1”.
Considering from all the above stages if we choose an edge e of G and examine the graph G’ = G – e.

If e is in a circuit, then G’ is connected any by the Induction Hypothesis. Hence we can observe that the conditions
r’ + n = (m-1) + 2, where f’ is the number of regions in G’.

In closing the circuit with e increase the number or regions by one so r’ = (r - 1) and the theorem (Euler’s Condition) is true.

If G-e is disconnected, then we can observe that it has two planar components, G1 and G2 whose number of vertices, edges and regions are n1, n2, m1, m2, f1 and f2 respectively.
By the Induction Hypothesis we can observe that

r1+ n1 = m1+2 and r2+ n2 = m2 +2

the number of regions becomes f1+ f2 – 1. The number of vertices becomes n1+ n2 , and the number of edges becomes m1+ m2+1.

By the above conditions it is true that in a planar embedding of a connected graph G has ‘n’ vertices, ‘m’ edges and ‘ r’ regions is true.

3.THE LINEAR BOUND CONDITION

“If a simple connected planar graph G has $n \geq 3$ vertices and m edges then $m \leq 3n-6$”.

In the region of the planar embedding of G are s1,s2,….sr, then we indicate the number of boundary edges of Si by riwhere (i=1,2,3…r).

The condition r=1 is obvious because G is a tree and $m=n-1 \leq 3n-6$. Similarly we assume that $r \geq 2$. Since G is simple, every region has at least 3 boundary edges and thusΣ ri $\leq$ 3m, (where i=1 to r).We can observe that from the above condition the the edge is a boundary edge of one or two regions in the planar embedding, so, Σ ri $\leq$ 2m, (where i=1 to r), this result now follows directly from Euler’s Polyhedron principal.

4.THE MINIMUM DEGREE BOND

For any Simple planar graph G, $\delta(G) \leq 5$.

A Characterization of planar graph is obtained by examining certain well defined conditions of sub graphs. If we consider the contradiction and counter hypothesis: G is a simple planar graph and , $\delta(G) \geq 6$. Then m. $\geq$ *3n, where n is the number of vertices and m is the number of edges in G.*

A graph is planar if and only if none of its subgraphscan be transformed to Kn where n=5 or Kn,nwehre n=3 by the corresponding edges.

There are many algorithms for testing planarity and drawing planar embedding. The concept of algorithm is to try to draw a graph on a plane step by step. If this does not exist then the graph is not planar. If G is a graph and R is a planar embedding of a planar sub graph S of G, then an R-part P of G is:

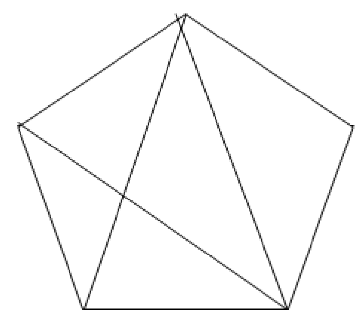
* Either an edge of G -S whose end vertices are in S.

* A component of the sub graph induced by vertices not in S which contains the edges that connect S to the component, known as pending edges, and their end vertices. Those vertices of an R-part of G that are end vertices of pending edges connecting them to S are called contact vertices. Usually we say that a planar embedding R of the planar sub graph S is planar extendable to G if R can be extended to a planar embedding of the whole G by drawing more vertices and edges. Such an extended embedding is called a planar extension of R to G. If G is planar, then at each step of the algorithm R is planar extendable to G.

If we use the condition of induction on the number of trials ‘l ‘ the algorithm visits the several conditions as it depending on the number of contact vertices of P, then Planar extends to R such that if P has at least two contact vertices v1 and v2 they are connected by a path p in P.If P has **exactly one** contact vertex v, with the corresponding pendant edge ‘E’, we apply Demoucrons Algorithm recursively with in P. If P has **no contact vertices**, we can use the Demoucron’s Algorithm recursively with input P. If it turns out that P is not planar, then G is not planar, and we output this information and stop. In extension we can use R to a planar embedding U by drawing P in one of its regions, set U to R. and return to its previous step.

5. K5 IS NON PLANAR

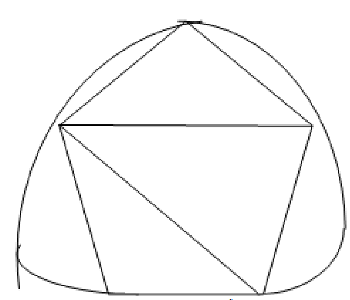
If we consider the applications of Graph theory then by contradiction. If possible let G be a plane graph corresponding to K5. Which denotes the vertices of G by v1,v2…..v5. Since G is Complete and two of its vertices are joined by an edge. In this process we can consider Jordan curve in the plane as Cycle which lies either in internal of Cycle ‘C’ or External of Cycle ‘C’.



G(V,E) Consisting of 5 Vertices and (2n-1)= 9 Edges.

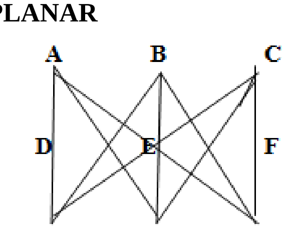
The above is Planar Graph G, then its embedding planar can be drawn as below for graph G’

Below is the form of Planar condition for the above Graph

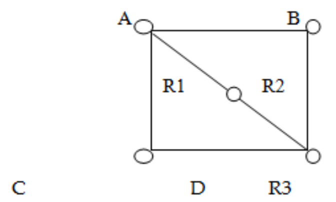


Graph G’

6. K3,3 IS NON PLANAR



If consider we start with a sub graph of K3,3, which isplanar. The following subgraph will partition the plane into here regions, R, R2, and R3



If Vertex b is put in R1 it can be connected to d and e with no edge crossings but there si no way to connect it to f with out a crossing. Similarly, if b is placed in R2, we must cross an existing edge to get to d. Finally, if b is placed in R3, we must cross and edge to get to e. Similar investigations with other sub graphs will show that there is no way to draw K3,3 is non planar.

7. 4-VERTEX COLOURABLE

Every planar graph is 4-vertex- colourable: A graph G is embeddable in the plane if any only if it is embeddable on the sphere.

In the above construction we can observe that two internal coincide edges are drawn externally to construct proper planar conditions. We can declare that he Graph G' is the planar illustrated condition of the given G.

In alternative state we can also consider the following method to prove that K_5 is Non planar:

Let us consider that V=5 so the condition of (3V-6) becomes 15-6=9. Hence Edges =C(5,2)=10 numbers

There fore E > 3V-6 and we conclude that K5 is non planner

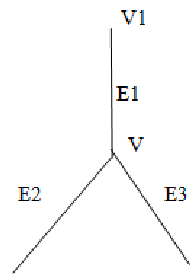
Let G has an embedding G on the sphere. Selecting a point z of the sphere not in G. Then the image of G under stereographic projection from z is an embedding of G in the plane. The converse of the condition is always true. By this it is clear that every planar graph is 4-vertex-colourable indicates that every plane graph is 4-face-colourable..

8. 4-FACE COLORABLE

Let us consider that every plane graph is 4-face colorable condition holds then G always simple 2-edge-connected 3-regular planar graph, and G be a planar embedding of G. By the condition of every plane graph is 4-face colorable then we can indicate that C0(00) , C1(1,0), C2(0,1), C3(1,1) are the corresponding vectors. Since G is 2 edge connected, each edge separates two distinct faces, and it follows that no edge is assigned the colour C0 under this condition. It is also clear that the three edges incident with a given vertex are assigned different colors.

9. 3-EDGE COLOURABLE

Let us consider that every simple 2-edge-connected 3-regular planar graph is 3-ede colorable. This always correspondences the condition that every planar graph is 4-vertex colorable. Hence 3-regular planar graph is always 3-edge colorable in every simple 2-edge connected 3-regular planar graph.



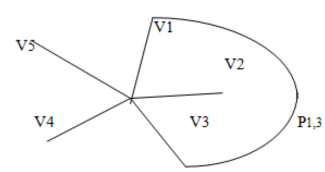
10. 5 -VERTEX COLOURABLE

If we consider that the Contradiction of the Condition "Every planar graph is 5-vertex-colourable." Then there exists a 6-critical plane graph G. Since a critical graph is simple, we see from corollary " If G is a simple planar graph then

$\delta(G) \leq 5$. But we know from the conditional statement and proof of Chromatic – If G is k-critical then $\delta \geq k-1$.

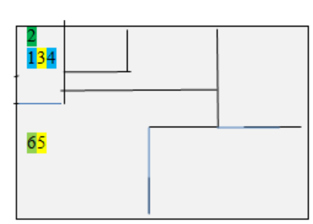
By the above two stages it is clear that $\delta = 5$. Let v be a vertex of degree five in G, and let (v1,V2,…V5) be a proper 5-vertex colouring of G – v; such a colouring exists because G is 6- critical, Since G itself is not 5-vertex colourable, v must be adjacent to a vertex of each of the five colours.

There fore we can assume that the neighbours of v in clockwise order about v are $V_1, V_2, \ldots V_5$ where V_i € V_i for $1 \leq i \leq 5$.



11. COLORING GRAPHS

Consider the Map below – with this we can test the conditions of fewest number of colors that will be sufficient to color any map in the plane. The major condition in Coloring graph is "the fewest number of colors are needed to color the map so that no to adjacent regions have the same color."



Example: Problem

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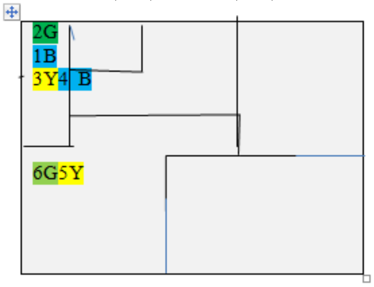
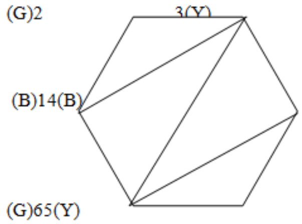
Two Colors are not sufficient, but we can do it with three colors only. We can observe the following colors in the Coloring Graph with Blue(1,4),Green(2,6), Yellow(3,5).



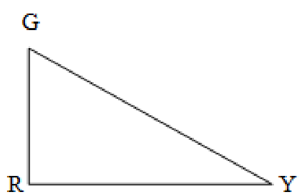
12. DUAL GRAPH OF THE MAPS



The Dual graph of a map is a connected simple graph with one vertex corresponding to each region and an edge

connecting two vertices if the corresponding regions on the map are as above.

The Coloring of asimple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices have the same color.The chromatic number of a graph G is the least number of colors needed for a coloring of the graph and is denoted X(G)



Chromatic Number X(G) = 3

The chromatic number of a planar graph is no greater than four. This statement is known as four colour Theorem.

Kn : vertex will need its own color, hence the Chromatic Number of Kn = n

K(m,n): The vertices of Km,n are partitioned into two sets with each edge connecting a vertex in one set to a vertex in the other set.

Thus, only two colors are needed, so Chromatic Number K(m,n) = 2.

CONCLUSION

The paper lays emphasis on the view point of Graph Theory Applications in Engineering modules. . We can observe that in the research process some fundamental properties of planar embedding of graphs. As a fundamental property we can observe that in any Planar Graph extension If a planar embedding of a connected graph G then it has n vertices, m edges and ' r' regions.

The chormatic number depends on the parity of n :ExampleCn: The Cycle Graph with n-vertices.

$\chi(C_n) = 1$, if n=1

$\chi(C_n) = 2$, if n is even

$\chi(C_n) = 3$, if n is odd and n > 1

In Dominator Chromatic Number with Ref.to S.M.Hedetnieni, S.T.Hedetniemi, A.A.Mc.Rae and JRS Blair, Dominator Coloring of Graphs will provide the relational cycle forms.

In General if we observe in a Planar graph degeneracy is Five(5). And A tree has degeneracy of One(1).

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